12TH INTERNATIONAL
BRICK/BLOCK
Masonry
CONFERENCE

Ade

MILLENNIUM ARCH. STRUCTURAL ANALYSIS

Josep Mª Adell[1], Concha del Rio[2]
[1]Professor, Dr Architect. UPM. Universidad Politécnica de Madrid. DCTA.
Architectural Building and Technology Department. ETSAM.
Escuela Técnica Superior de Arquitectura.
[2]Professor, Dr Architect. UPM. Universidad Politécnica de Madrid. DCTA.
Architectural Building Structures. ETSAM. Escuela Técnica Superior de Arquitectura.

STRENGTH ARRANGEMENT

The strength arrangement of the "Millennium Arch" may be seen in the form of a cantilever arch beam spanning some 11m and cantilevering 2.50 m over the support, embedded at each end in reinforced masonry walls, which form a cylindrical surface set at a 30º angle to the vertical.

The load state is essentially made up of the deadweight of the structure, though for load analysis and sizing purposes, consideration has also been given to a possible simultaneous live load as a result of usage or snow and due to the form of the "enclosure" framework, which may be observed in some of the stress diagrams shown.

As a result of this structural arrangement, the Millennium Arch is essentially subject to bending, torsional and shear stress in the area of the cantilever beam element and to predominantly compressive stress at the support walls, where there will also be tensile stresses at the point where the beam is housed due to the strong counteraction at the end of the same in order to balance the beam.

It is necessary to indicate that in spite of the fact that the structure is extraordinarily redundant, the main stresses in the analysis refer to ones of equilibrium with very little margin of variation in accordance with the compatibility conditions imposed.

ANALYSIS MODELS

The analysis of the structure was carried out in two successive stages, using a different model in each. These two stages correspond to two different areas of approximation of the problem, required to control, on the one hand, the basic stresses of bending and torsion in the arch beam and, on the other, the effect that the housing of the beam has on the masonry supports, particularly in the areas of reaction.

A model was first made to analyse the arch beam, which consists of a curved bar of 29x100cm with an elliptical directrix, the flat plane of which being angled to the direction of loading. The bar has been divided into three equal segments connected by rigid joints. The conditions imposed at the ends correspond to those of "perfect housing", that is to say, total resistance to overturning and movement. The load state corresponds to gravitational action due to the deadweight when taking a material density of 18kN/m? and an eventual live load of 3knN/ml. The stress-strain diagrams enclosed have been obtained on the basis of this model and when taking an elastic and lineal behaviour, and the maximum values have served to verify the predimensioning of the reinforcement and the minimum spacing of the transverse reinforcement.

The second model of analysis corresponds to the complete arch and was carried out with the aim of localizing and evaluating the presence of tensile stress in the areas where the beam connects with the support walls. The analysis also served to estimate the instantaneous strain at the keystone and in order to establish a slight bow during the construction stage to improve the visual appearance. As such, the complete structure was arranged in the form of a dense mass of bars, referred to in the diagrams as "bed-joint bars" and "joint bars", which closely follow the line of the reinforcement and which are interconnected by sufficiently rigid diagonal connecting rods, which simulate the framework assembly. The load state is similar to that of the previous model. The stresses obtained by this second model also provide greater information regarding the optimum distribution of the flexural reinforcement of the beam.

Successive partial structures were analysed during the construction of the arch, with each model segment corresponding to the different stages of construction. These simpler models were employed to verify the equilibrium of the uncompleted structure and to establish the shoring required during construction. It should be noted that the stress distribution noticeably differed from that indicated for the completed structure and, therefore, the reinforcement arrangement obviously had to cover the requirements of the building stage though this, subsequently, means that the reinforcement is somewhat oversized in some areas with regards to the working requirements of the completed structure.

DIMENSIONING MODELS

From the stress diagrams obtained, we then proceeded to verify the amount of reinforcement established in areas subject to most stress.

The sections subject to bending were scaled by means of a model similar to that of reinforced concrete, and taking a plastic behaviour of the failure section, limiting the maximum safe compressive stress in an equivalent rectangular block as 0.5kN/cm? and limiting the depth of the compressed block to half the section edge.

The ties were dimensioned by obtaining the maximum tensile stress through torsion and shear, and trusting that the reinforced masonry would uniformly develop a strength capacity by oblique compression similar to that of reinforced concrete. In spite of the fact that the torsion dimensioning procedure indicated in the Spanish Code EHE for reinforced concrete was followed, it was impossible to respect the minimum distance imposed between tie bars according to section diameter, as the available distance (15cm) is conditioned by the position of the voids in the brickwork which house the bars. However, not even the slightest indications of torsional cracking have been noted in the brickwork to date.

The vertical reinforcement placed in the side walls resist the local tensile stresses provoked by the strong reaction of the end of the beam in order to balance the same and also serve to supply the necessary tying to allow the safe raising and transport of the arch. As such the vertical reinforcement has been uniformly arranged throughout the side walls at relatively small spacings.

RESULTS

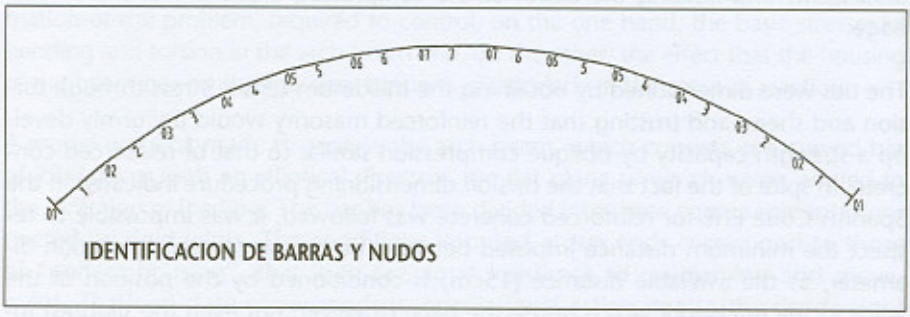
The scale stress diagrams obtained for each analytical model described are attached together with the most significant values obtained in the same.

A computer programme was used which allowed the analysis of any spatial bar structure by means of a matrix procedure. The programme had been designed for the analysis and dimensioning of concrete of steel elements and it was, therefore, only necessary to modify the parameters related to strength capacity and stiffness in order to suit the material employed.

While the stressing of all the bars forming the model have been indicated, it is obvious that the most significant figures are those corresponding to the most stressed areas within the line of the arch beam and particularly at the point where this meets the lateral support walls.

When comparing the diagrams for the two analysed models it may be seen that while there is a fundamental difference in the support conditions for the beam in both models, the flexural and torsional stress is not significantly different. It may then be predicted that the breaking stress will not differ much from the elastic stage, at least in the most stressed areas and, therefore, a failure test on this structure would provide sufficient information to know the stress state and specific behaviour of the new material under investigation.

MODELO PARA EL ANALISIS
Proyecto: Arco del Milenio (ARC-MIL)
Estructura: Viga curva (V-C1)



IDENTIFICACION DE BARRAS Y NUDOS

Listado de solicitaciones de cálculo (γf = 1,6)
Proyecto: Arco del milenio (ARC-MIL)
Estructura: Viga curva (V-C1)

Barra	NN	x (cm)	HIP	Mx (m.kN)	My	Mz	Fx (kN)	Vy	Vz
1	01	0	M+	+5,3	+3,9	+00,0	+00,0	+00,0	+2,9
1	02	84	M+	+5,3	+1,5	+00,0	+00,0	+00,0	+2,9
1	01	0	M-	+00,0	+00,0	-131,1	-25,6	-60,9	+00,0
1	02	84	M-	+00,0	+00,0	-84,0	-22,3	-51,4	+00,0
2	02	0	M+	+00,0	+00,0	+00,0	+00,0	+00,0	+2,5
2	03	85	M+	+00,0	+00,0	+00,0	+00,0	+00,0	+2,5
2	02	0	M-	-3,7	-4,4	-84,8	-19,7	-52,4	+00,0
2	03	85	M-	-3,7	-2,2	-43,6	-16,9	-42,6	+00,0
3	03	0	M+	+00,0	+3,2	+00,0	+00,0	+00,0	+2,0
3	04	84	M+	+00,0	+1,5	+00,0	+00,0	+00,0	+2,0
3	03	0	M-	-8,7	+00,0	-42,8	-15,1	-43,3	+00,0
3	04	84	M-	-8,7	+00,0	-10,5	-12,7	-33,5	+00,0
4	04	0	M+	+00,0	+1,2	+00,0	+00,0	+00,0	+1,5
4	05	84	M+	+00,0	+00,0	+15,2	+00,0	+00,0	+1,5
4	04	0	M-	-9,8	+00,0	-9,3	-10,8	-34,1	+00,0
4	05	84	M-	-9,8	-0,1	+00,0	-9,0	+24,2	+00,0
5	05	0	M+	+00,0	+00,0	+16,3	+00,0	+00,0	+1,0
5	06	84	M+	+00,0	+00,0	+32,7	+00,0	+00,0	+1,0
5	05	0	M-	-8,0	-0,9	+00,0	-7,7	-24,6	+00,0
5	06	84	M-	-8,0	-1,7	+00,0	-6,5	-14,6	+00,0
6	06	0	M+	+00,0	+2,4	+33,4	+00,0	+00,0	+0,5
6	07	84	M+	+00,0	+2,8	+41,6	+00,0	+00,0	+0,5
6	06	0	M-	-4,6	+00,0	+00,0	-5,7	-14,9	+00,0
6	07	84	M-	-4,6	+00,0	+00,0	-5,1	-4,8	+00,0
7	07	0	M+	+00,0	+00,0	+41,9	+00,0	+00,0	+00,0
7	08	84	M+	+00,0	+00,0	+41,9	+00,0	+5,0	+00,0
7	07	0	M-	-00,0	-3,1	+00,0	-4,9	-5,0	-00,0
7	08	84	M-	-00,0	-3,1	+00,0	-4,9	+00,0	-00,0

DIAGRAMAS DE ESFUERZOS CORTANTES Y AXILES
Proyecto: Arco del Milenio (ARC-MIL)
Estructura: Viga curva (V-C1)

ESFUERZOS CORTANTES "Y"
Valor máximo: 60,9 kN

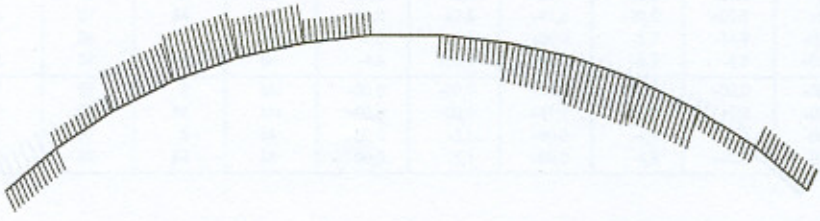
ESFUERZOS CORTANTES "Z"
Valor máximo: 2,9 kN

ESFUERZOS AXILES
Valor máximo: 25,6 kN

DIAGRAMAS DE MOMENTOS FLECTORES Y TORSORES
Proyecto: Arco del Milenio (ARC-MIL)
Estructura: Viga curva (V-C1)

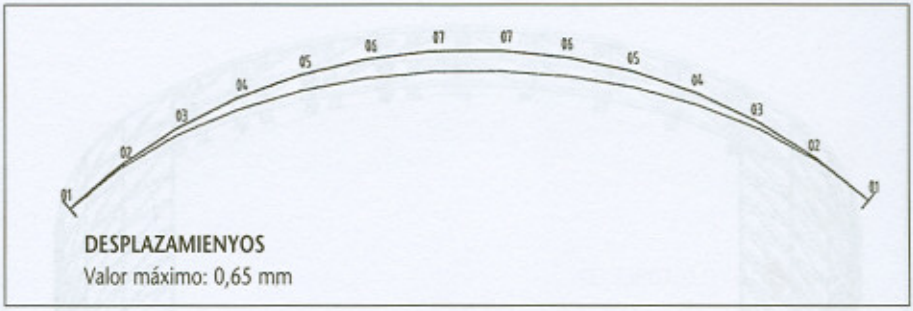
MOMENTOS FLECTORES "Z"
Valor máximo: 131,1 m.kN

MOMENTOS FLECTORES "Y"
Valor máximo: 4,4 m.kN



MOMENTOS TORSORES
Valor máximo: 9,8 m.kN

DIAGRAMA DE DESPLAZAMIENTOS
Proyecto: Arco del Milenio (ARC-MIL)
Estructura: Viga curva (V-C1)



DESPLAZAMIENYOS
Valor máximo: 0,65 mm

Listado de desplazamientos de nudos
Proyecto: Arco del milenio (ARC-MIL)
Estructura: Viga curva (V-C1)

NN	Tipo	Dx (mm)	Dy (mm)	Dz (mm)	Gx (rad)	Gy (rad)	Gz (rad)
01	fijo	+0,00	+0,00	+0,00	+0,0000	+0,0000	+0,0000
02	libre	-0,00	-0,05	-0,02	-0,0000	-0,0000	+0,0000
03	libre	-0,01	-0,16	-0,08	-0,0001	-0,0001	+0,0000
04	libre	-0,01	-0,32	-0,16	-0,0002	-0,0001	+0,0000
05	libre	-0,00	-0,47	-0,26	-0,0003	-0,0001	+0,0000
06	libre	-0,00	-0,59	-0,34	-0,0003	-0,0000	+0,0000
07	libre	-0,00	-0,65	-0,38	-0,0004	-0,0000	+0,0000

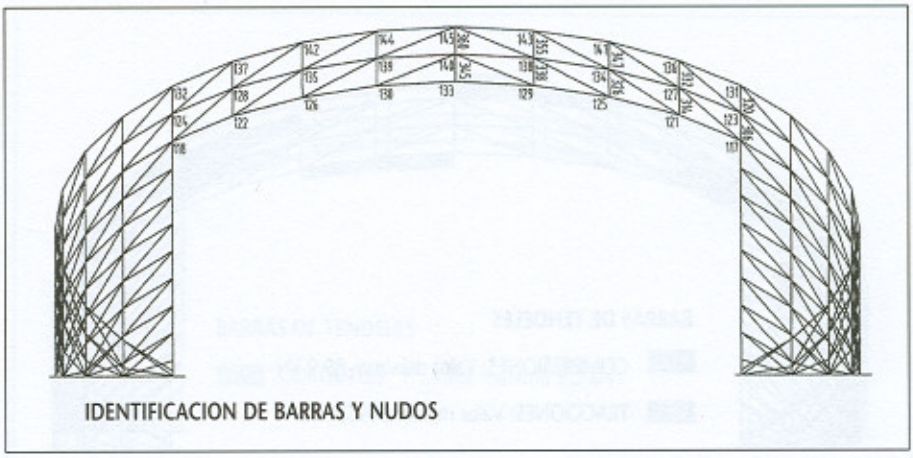
MODELO PARA EL ANALISIS
Proyecto: Arco del Milenio (ARC-MIL)
Estructura: Arco completo (A-C1)

IDENTIFICACION DE BARRAS Y NUDOS

Listado de solicitaciones de cálculo (γf = 1,6)
Barra de tendeles
Proyecto: Arco del milenio (ARC-MIL)
Estructura: Arco completo (A-C1)

Barra	NN	x (cm)	Mx (m.kN)	My	Mz	Fx (kN)	Vy	Vz
343	131	0	-1,4	-7,2	-4,4	+92,2	-9,5	+7,0
343	136	137	-1,4	+2,5	+1,3	+95,7	+1,2	+7,0
352	136	0	-3,5	-4,9	-2,7	+32,6	-7,4	+7,7
352	141	137	-3,5	+5,6	+0,0	+35,3	+3,5	+7,7
361	141	0	-3,7	-0,5	-1,3	-42,1	-5,7	+3,6
361	143	136	-3,7	+4,4	-1,1	-40,6	+5,4	+3,6
363	143	0	-1,8	+1,9	-0,4	-89,9	-5,0	+0,1
363	145	137	-1,8	+2,0	-1,3	-89,4	+6,3	+0,1
319	123	0	-1,8	+2,8	-4,1	+45,0	-6,0	+3,6
319	127	137	-1,8	-2,2	+1,7	+46,1	-2,6	+3,6
331	127	0	-3,8	+0,6	-3,4	+10,4	-5,7	+2,7
331	134	137	-3,8	-3,1	+1,9	+11,2	-2,1	+2,7
346	134	0	-3,5	+0,6	-2,0	-0,5	-4,4	+0,5
346	136	136	-3,5	+1,3	+1,5	+0,0	-0,8	+0,5
354	138	0	-1,4	+0,9	-0,3	-0,5	-2,7	-1,5
354	140	137	-1,4	-1,1	+0,9	-0,3	+0,9	-1,5
305	117	0	-1,4	+2,3	-7,8	-87,6	-8,8	+4,7
305	121	137	-1,4	-4,1	+1,9	-86,5	-5,4	+4,7
313	121	0	-3,7	-4,2	-3,1	-3,8	-5,0	-3,7
313	125	137	-3,7	+0,9	+1,4	-3,0	-1,5	-3,7
325	125	0	-3,0	+2,6	-2,5	+42,0	-5,5	-4,0
325	129	136	-3,0	-2,9	+2,6	+42,5	-2,0	-4,0
337	129	0	-0,9	-0,1	-0,5	+49,9	-3,4	-2,1
337	133	137	-0,9	-3,0	+1,7	+50,1	+0,2	-2,1

MODELO PARA EL ANALISIS
Proyecto: Arco del Milenio (ARC-MIL)
Estructura: Arco completo (A-C1)

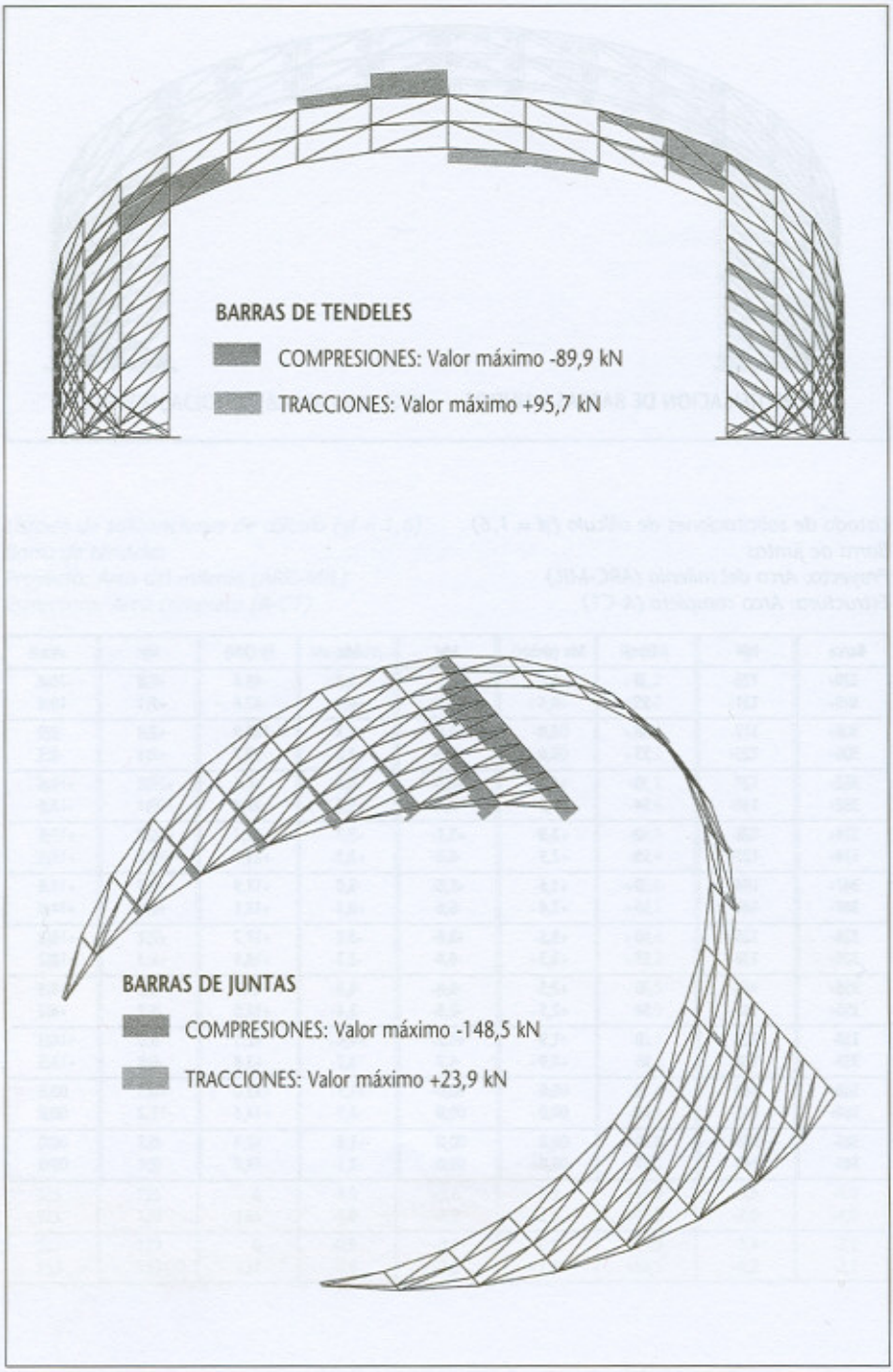


IDENTIFICACION DE BARRAS Y NUDOS

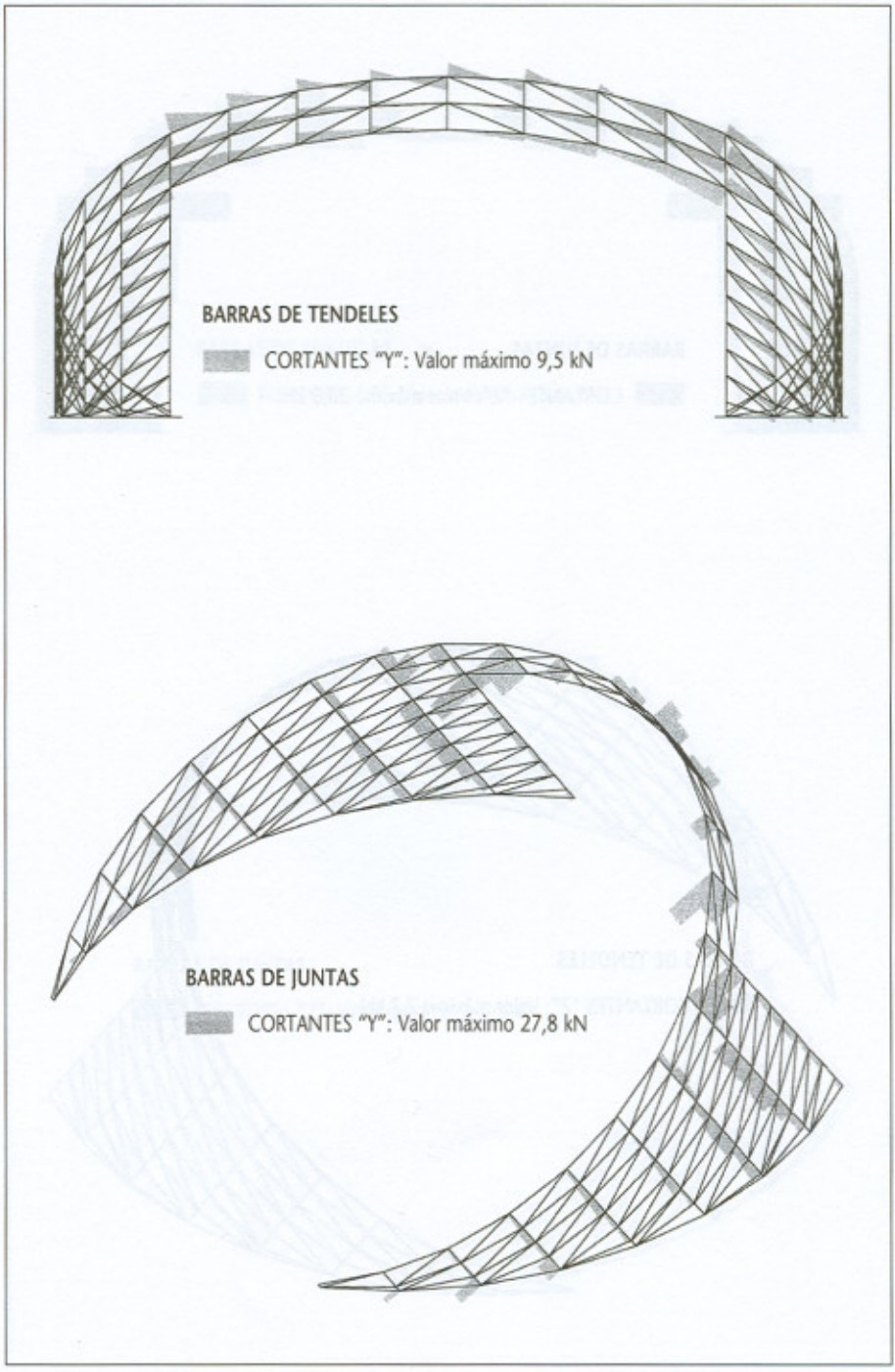
Listado de solicitaciones de cálculo (γf = 1,6)
Barra de juntas
Proyecto: Arco del milenio (ARC-MIL)
Estructura: Arco completo (A-C1)

Barra	NN	x (cm)	Mx (m.kN)	My	Mz	Fx (kN)	Vy	Vz
320	123	0	-0,4	-3,7	-3,2	-48,8	+7,8	-10,6
320	131	55	-0,4	+2,1	+0,9	-47,6	+7,1	-10,6
306	117	0	00,0	-13,9	-2,3	-100,9	+2,8	-2,3
306	123	55	00,0	-1,3	-1,0	-99,7	+2,1	-2,3
332	127	0	+2,5	+2,8	-9,4	+6,6	+27,8	+18,6
332	136	54	+2,5	-7,2	+5,5	+7,8	+27,1	+18,6
314	121	0	+2,9	+3,1	-7,3	+22,7	+16,0	+17,1
314	127	55	+2,9	-6,3	+1,3	+23,9	+15,2	+17,1
347	134	0	+3,6	+1,5	-5,0	+17,9	+9,7	+14,6
347	141	55	+3,6	-6,6	+0,1	+19,1	+9,0	+14,6
326	125	0	+3,5	+3,6	-3,2	+17,2	+2,1	+18,2
326	134	55	+3,5	-6,4	-2,3	+18,5	+1,3	+18,2
355	138	0	+2,5	-0,4	-0,9	+9,7	-4,2	+4,3
355	143	54	+2,5	-2,8	-3,4	+11,0	-4,9	+4,3
338	129	0	+1,9	+3,2	+0,4	+2,5	-6,0	+14,3
338	138	55	+1,9	-4,7	-3,2	+3,8	-6,8	+14,3
360	140	0	00,0	00,0	+1,9	+13,0	-10,5	00,0
360	145	54	00,0	00,0	-3,9	+14,3	-11,2	00,0
345	133	0	00,0	00,0	+1,8	+2,4	-6,7	00,0
345	140	55	00,0	00,0	-2,1	+3,7	-7,4	00,0

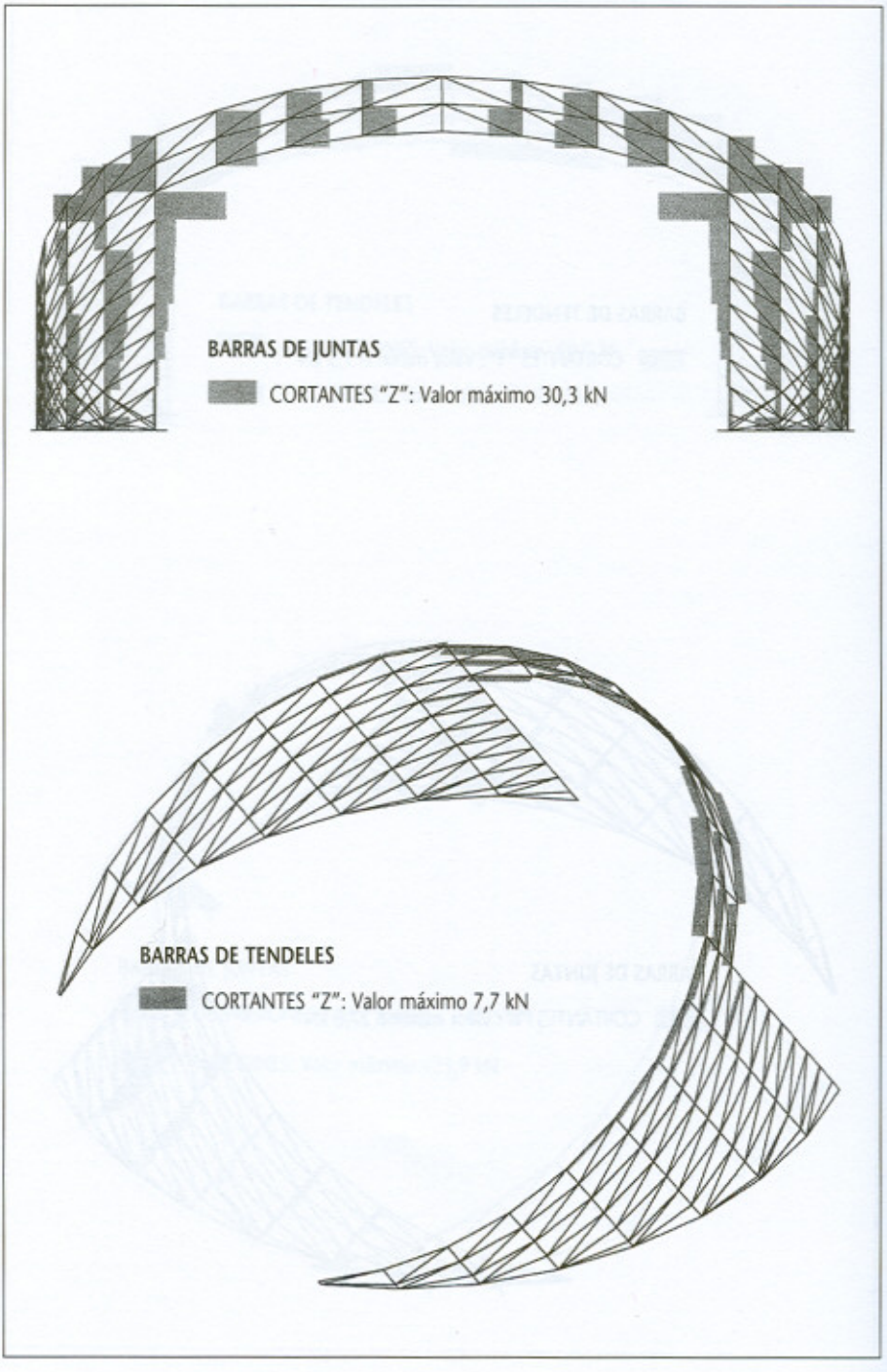
DIAGRAMAS DE ESFUERZOS AXILES
Proyecto: Arco del Milenio (ARC-MIL)
Estructura: Arco completo (A-C1)



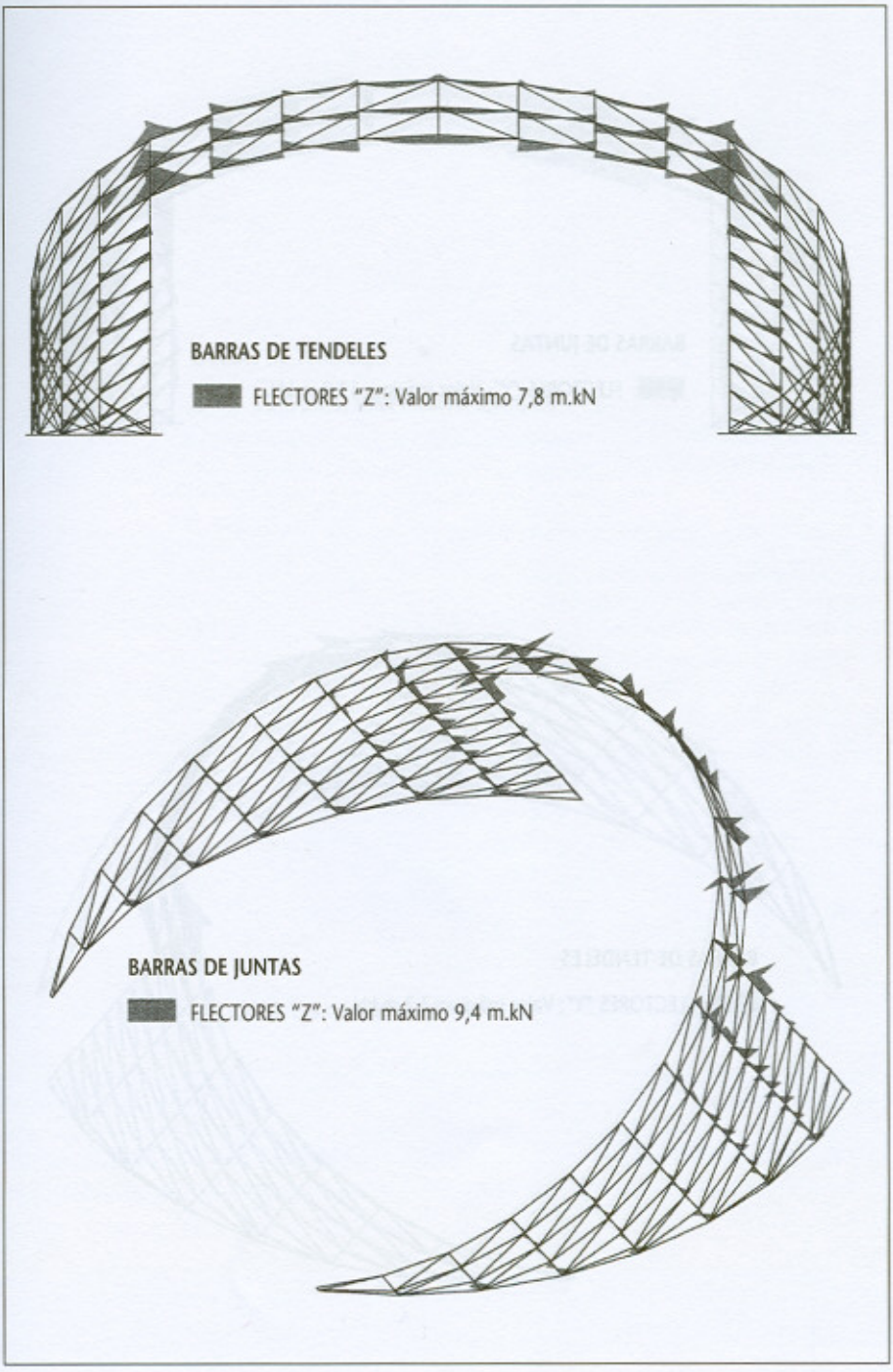
DIAGRAMAS DE ESFUERZOS CORTANTES "Y"
Proyecto: Arco del Milenio (ARC-MIL)
Estructura: Arco completo (A-C1)



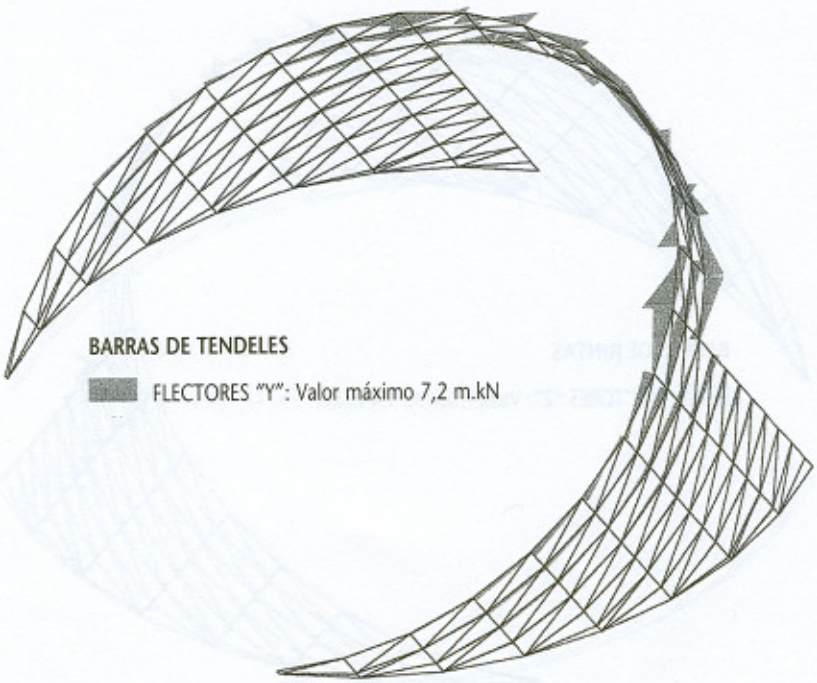
DIAGRAMAS DE ESFUERZOS CORTANTES "Z"
Proyecto: Arco del Milenio (ARC-MIL)
Estructura: Arco completo (A-C1)



DIAGRAMAS DE MOMENTOS FLECTORES "Z"
Proyecto: Arco del Milenio (ARC-MIL)
Estructura: Arco completo (A-C1)



DIAGRAMAS DE MOMENTOS FLECTORES "Y"
Proyecto: Arco del Milenio (ARC-MIL)
Estructura: Arco completo (A-C1)



DIAGRAMAS DE MOMENTOS TORSORES
Proyecto: Arco del Milenio (ARC-MIL)
Estructura: Arco completo (A-C1)

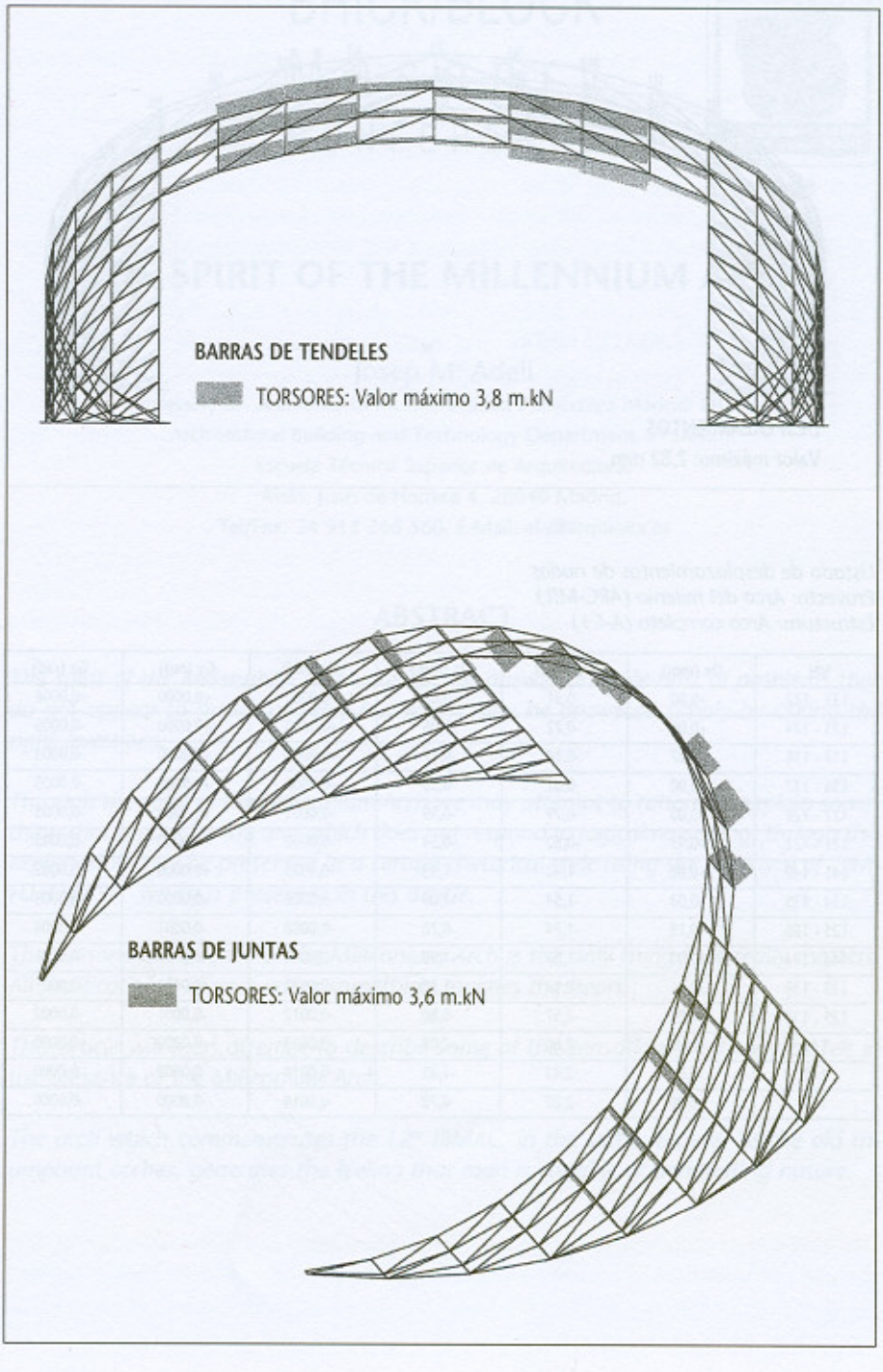
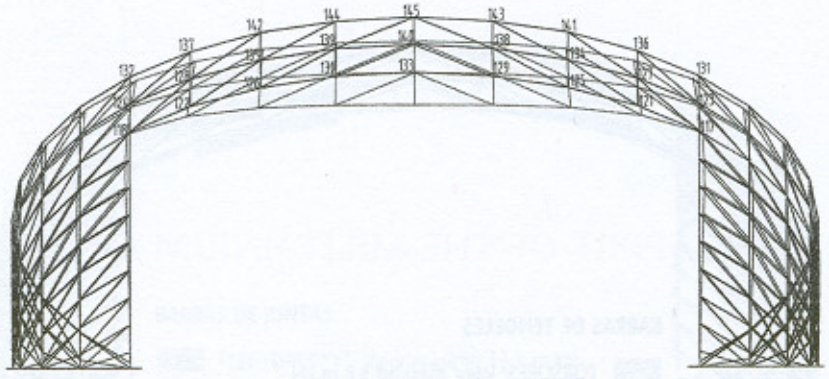


DIAGRAMA DE DESPLAZAMIENTOS
Proyecto: Arco del Milenio (ARC-MIL)
Estructura: Arco completo (A-C1)



DESPLAZAMIENTOS
Valor máximo: 2,82 mm

Listado de desplazamientos de nudos
Proyecto: Arco del milenio (ARC-MIL)
Estructura: Arco completo (A-C1)

NN	Dx (mm)	Dy (mm)	Dz (mm)	Gx (rad)	Gy (rad)	Gz (rad)
131 - 132	+0,00	-0,31	+0,00	+0,0000	+0,0000	+0,0006
123 - 124	+0,00	-0,22	-0,08	+0,0000	+0,0000	-0,0006
117 - 118	-0,27	-0,14	-0,17	+0,0000	+0,0000	-0,0003
136 - 137	+0,00	-0,81	-0,55	+0,0000	+0,0000	-0,0005
127 - 128	-0,00	-0,79	-0,59	-0,0001	+0,0000	-0,0005
121 - 122	-0,32	-0,82	-0,54	-0,0002	+0,0000	-0,0005
141 - 142	+0,00	-1,40	-1,33	-0,0005	+0,0000	-0,0002
134 - 135	-0,03	-1,54	-1,09	-0,0006	+0,0000	-0,0003
125 - 126	-0,18	-1,74	-0,76	-0,0008	-0,0001	-0,0004
143 - 144	+0,00	-1,87	-1,90	-0,0010	+0,0000	-0,0001
138 - 139	-0,01	-2,17	-1,38	-0,0012	+0,0000	-0,0001
129 - 130	-0,05	-2,51	-0,80	-0,0012	-0,0001	-0,0002
145	-0,00	-2,06	-2,09	-0,0013	-0,0000	+0,0000
140	-0,00	-2,43	-1,45	-0,0014	-0,0000	-0,0000
133	-0,00	-2,82	-0,78	-0,0014	-0,0000	-0,0000